Statistics
Fall 2022
Lecture 8

Binomial Prob. Dist.: (SG 16)

1) n independent events (trials)

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $\quad$ $P(\text{Failure}) = q$

3) p & q remain unchanged for all events.

$p + q = 1$, $q = 1 - p$

4) x is # of successes , $x \geq 0$

$x = 0, 1, 2, 3, \ldots$

$$\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$$

5) Mean $\mu = np$

Variance $\sigma^2 = npq$

Standard deviation $\sigma = \sqrt{\sigma^2}$

Consider a binomial Prob. dist. with $n = 12$, and $p = .6$.

1) $q = 1 - p = 1 - .6 = \boxed{.4}$

2) $\mu = np = 12(.6) = \boxed{7.2}$

3) $\sigma^2 = npq = 12(.6)(.4) = \boxed{2.88}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{2.88} \approx \boxed{1.697}$

Let x be # of Successes

5) P(exactly 8 Successes)

$= P(x = 8) = {}_{12}C_8 \cdot (.6)^8 \cdot (.4)^4 = \boxed{.213}$ ✓

${}_nC_x \cdot p^x \cdot q^{n-x}$

Suppose we flip a fair coin 20 times, and landing tails is a Success.

1) $n = 20$

2) $p = .5$

3) $q = .5$

4) $\mu = np$
$= 20(.5)$
$= \boxed{10}$

5) $\sigma^2 = npq$
$= 20(.5)(.5)$
$= \boxed{5}$

6) $\sigma = \sqrt{\sigma^2}$
$= \sqrt{5}$
$\approx \boxed{2.236}$

Let x be # of tails

5) P(land exactly 12 tails)

$= P(x = 12) = {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^8 = \boxed{.120}$

${}_nC_x \cdot p^x \cdot q^{n-x}$

Now using TI Command

[2nd] [VARS] [↓] [binompdf(] $\overset{n}{20}, \overset{p}{.5}, \overset{x}{12}$) [Enter]

Trials: 20

p: .5

x-value: 12

[Paste]

Consider a multiple-choice exam with $\boxed{40 \text{ questions}}$, and we are making random guesses.

Suppose each question has $\boxed{4 \text{ choices}}$ but only $\boxed{1 \text{ correct choice}}$. Success is to guess Correctly.

1) $n = 40$

2) $p = \frac{1}{4} = \boxed{.25}$

3) $q = \frac{3}{4} = \boxed{.75}$

4) $\mu = np = 40(.25) = \boxed{10}$

5) $\sigma^2 = npq = 40(.25)(.75) = \boxed{7.5}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{7.5} \approx \boxed{2.739}$

Round μ & σ to a whole #, Find

$\mu \to \boxed{10}$, $\sigma \to \boxed{3}$

7) 68% Range

$\mu \pm \sigma = 10 \pm 3 \Rightarrow \boxed{7 \text{ to } 13}$

8) Usual Range

95% Range

$\mu \pm 2\sigma = 10 \pm 2(3) \Rightarrow \boxed{4 \text{ to } 16}$

P(guess exactly 15 correct ans.)

n, P, x

$= P(x = 15) = \text{binompdf}(40, .25, 15)$

$= \boxed{.028}$

P(guess at most 15 Correct ans.)

$= P(x \leq 15) = P(x=15) + P(x=14) + P(x=13) + \dots + P(x=0)$

$= \text{binomcdf}(40, .25, 15)$

$= \boxed{.974}$

Consider a binomial Prob. dist. with $n=400$, and $P=.8$

1) $q=1-P=\boxed{.2}$

2) $\mu=np=\boxed{320}$

3) $\sigma^2=npq=\boxed{64}$

4) $\sigma=\sqrt{\sigma^2}=\boxed{8}$

5) 68% Range $\Rightarrow \mu \pm \sigma$

$=\boxed{312 \text{ to } 328}$

6) $\underbrace{\text{95\% Range}}_{\text{usual Range}} \Rightarrow \mu \pm 2\sigma$

$=\boxed{304 \text{ To } 336}$

7) P(exactly 325 Successes)

$= P(x=325) = \text{binompdf}(400, .8, 325) = \boxed{.042}$ ✓

8) P(fewer than 330 successes)

$= P(x<330) = P(x \leq 329) = \text{binomcdf}(400, .8, 329)$

$= \boxed{.884}$ ✓

9) P(at least 310 Successes)

$= P(x \geq 310) = 1 - P(x \leq 309)$

$= 1 - \text{binomcdf}(400, .8, 309)$

we don't want | we want

309 310

$= \boxed{.904}$

100 newborn babies were randomly selected.

Suppose success is having a girl,

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = \boxed{50}$ 5) $\sigma^2 = npq = \boxed{25}$ 6) $\sigma = \sqrt{\sigma^2} = \boxed{5}$

7) Usual Range $\Rightarrow \mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

8) P(exactly 55 girls) $= P(X = 55) = \text{binompdf}(100, .5, 55) = \boxed{.048}$

9) P(at most 60 girls) $=$

$P(X \leq 60) = \text{binomcdf}(100, .5, 60) = \boxed{.982}$

10) P(at least 45 girls) $=$

$= P(X \geq 45) = 1 - P(X \leq 44) = 1 - \text{binomcdf}(100, .5, 44)$

we don't want 44 | 45 we want $= \boxed{.864}$

$$P(a \leq X \leq b) = \text{binomcdf}(n, P, b) - \text{binomcdf}(n, P, a-1)$$

Reduce by 1

P(between 40 and 60, inclusive, girls)

$$P(40 \leq X \leq 60) = \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$$

Reduce by 1

$= \boxed{.965}$

Prob. of any student likes Zoom classes is $\frac{1}{3}$.
Suppose we randomly select 60 students.
Success is to like Zoom classes.

1) $n = 60$ 2) $p = \frac{1}{3}$ 3) $q = \frac{2}{3}$

4) $\mu = np = \boxed{20} \rightarrow 20$ 5) $\sigma^2 = npq = \frac{40}{3}$ 6) $\sigma = \sqrt{\sigma^2} \approx \boxed{3.651} \rightarrow 4$

Round μ & σ to whole #, then find

7) 68% Range $\Rightarrow \mu \pm \sigma \Rightarrow \boxed{16 \text{ to } 24}$

8) P(between 16 to 24, inclusive, like Zoom Classes)

$P(16 \le x \le 24) = \text{binomcdf}(60, 1/3, 24) - \text{binomcdf}(60, \frac{1}{3}, 15) = \boxed{.783}$

Reduce by 1

SG 16 ✓

Geometric Prob. Dist: SG 17

It is similar to binomial Prob. dist except there is no n.

$p \rightarrow$ Prob. of success
$q \rightarrow$ Prob. of failure

$p + q = 1$, $q = 1 - p$
p & q remain unchanged for any trials.

x is # of trial where first success happens
$x = 1, 2, 3, 4, \ldots$

$$P(x) = p \cdot q^{x-1}$$

$$\mu = \frac{1}{p}, \quad \sigma^2 = \frac{q}{p^2}, \quad \sigma = \sqrt{\sigma^2}$$

Consider a geometric Prob. dist. with $p = .6$

$q = 1 - p = .4$

$\mu = \frac{1}{p} = \frac{1}{.6} = 1.667$

$\sigma^2 = \frac{q}{p^2} = \frac{.4}{.6^2} = 1.111$

$\sigma = \sqrt{\sigma^2} = \sqrt{1.111} \approx 1.054$

P(First Success happens at 2nd trial)

$P(X = 2) = (.6)(.4)^{2-1} = \boxed{.24}$

$p \cdot q^{x-1}$

$\boxed{2nd}$ $\boxed{VARS}$ $\boxed{\downarrow}$ $\boxed{\text{geomet pdf}}(.6, 2) = \boxed{.24}$

P(First Success happens before 3rd trial)

$P(X < 3) = P(X = 1) + P(X = 2)$

$= \text{geometcdf}(.6, 2) = \boxed{.84}$

LeBron makes 80% of his F.T.

$p = .8$ $q = .2$

$\mu = \frac{1}{p} = \frac{1}{.8} = \boxed{1.25}$

$\sigma^2 = \frac{q}{p^2} = \frac{.2}{.8^2} = \boxed{.3125}$

$\sigma = \sqrt{\sigma^2} = \sqrt{.3125} \approx \boxed{.559}$

P(He makes his First FT on 4th attempt)

$P(X=4) = \text{geomet pdf}(.8, 4) = \boxed{.006}$

P(He makes his First FT before 4th attempt)

$P(X<4) = P(X \leq 3)$

$= \text{geomet cdf}(.8, 3) = \boxed{.992}$

P(He makes his First FT after 4th attempt)

$P(X>4) = P(X \geq 5) = 1 - P(X \leq 4)$

we don't want

we want

$= 1 - \text{geometcdf}(.8, 4)$

4 5

.0016

$= \boxed{.002}$

Poisson Prob. dist.

SG 17

Average # of Successes in a fixed interval is given $\Rightarrow$ μ or λ

Let x be # of Successes in that interval

$$P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$$

$x = 0, 1, 2, 3, \ldots$

$e \approx 2.718$

$\sigma^2 = \mu$ $\qquad$ $\sigma = \sqrt{\sigma^2}$

Consider a Poisson Prob. dist with $\mu = 9$ in a fixed interval.

$\sigma^2 = \mu = 9$ $\qquad$ $\sigma = \sqrt{\sigma^2} = \sqrt{9} = 3$

Usual Range $\Rightarrow \mu \pm 2\sigma = 9 \pm 2(3)$

$\Rightarrow \boxed{3 \text{ to } 15}$

$P(10 \text{ Successes}) = P(x = 10)$

$= \text{Poisson Pdf}(\underbrace{9}_{\mu \text{ or } \lambda}, 10) = \boxed{.119}$

P(at most 15 Successes) =

$P(x \leq 15) = \text{Poisson cdf}(9, 15) = \boxed{.978}$

P(at least 5 Successes)

$P(x \geq 5) = 1 - P(x \leq 4)$

$= 1 - \text{Poissoncdf}(9, 4)$

$= \boxed{.945}$

we don't want 4

we want 5

In average, Mr. Xu gets 10 complaints per day.

$\mu = 10$

$\lambda = 10$

fixed interval

P(He gets fewer than 10 complains per day)

$P(x < 10) = P(x \le 9)$

$= \text{poissoncdf}(10, 9)$

$= \boxed{.458}$

P(he gets between 8 and 12 complaints, inclusive)

$P(8 \le x \le 12)$

$\text{Poissoncdf}(10, 12) - \text{Poissoncdf}(10, 7)$

$= \boxed{.571}$

In average, I have 4 students, tardy or absent, per class meeting. $\rightarrow \mu = 4$

Fixed Interval

$\sigma^2 = \mu = \boxed{4}$ $\sigma = \sqrt{\sigma^2} = \sqrt{4} = \boxed{2}$

68% Range $\Rightarrow \mu \pm \sigma \Rightarrow \boxed{2 \text{ to } 6}$

P(I have exactly 10 students, tardy/absent)

$P(X = 10) = \text{Poisson Pdf}(4, 10)$

$= \boxed{.005}$

P(at most 8 tardy/absent)

$P(x \le 8) = \text{Poisson Cdf}(4, 8) = \boxed{.979}$

P(at least 2 tardy/absent)

$P(x \ge 2) = 1 - P(x \le 1) = 1 - \text{Poissoncdf}(4, 1)$

$= \boxed{.908}$

we don't want | we want

1 2

SG 17

watch the videos on the right of SG 18 to 21.